5. Economics of networks

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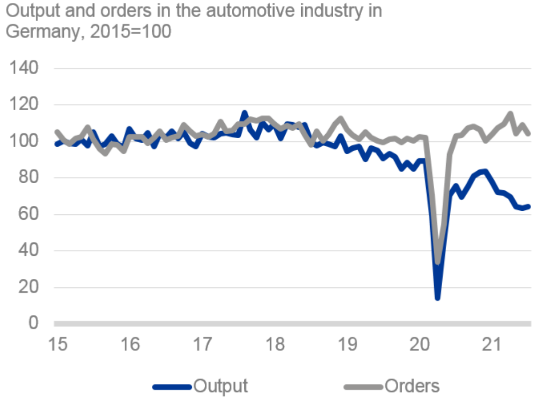
Shortages in the automobile industry



Source: Deutsche Bank

Forbes

LEADERSHIP STRATEGY

Supply Chain Economics: Car Chip Shortage

Bill Conerly Senior Contributor
I connect the dots between the economy … and business!

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Bosch's new semiconductor factory (Photo by Robert Michael/picture alliance via Getty Images)
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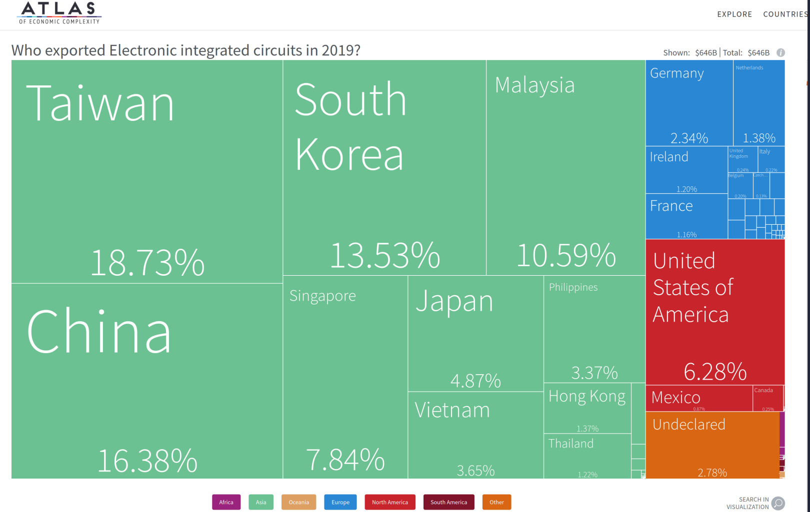

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Supply chain of mobile phone handsets before 2009-2012

Notes: Network of the GVCs of smartphones between 2009 and 2012. Network representation of the total amount exchanged between/within countries for all mayor brands of smartphones. The shade of the links between countries as well as their widths are proportional to the total amount of firms exchanging between countries. The darker the color, the greater the number of firm-transactions. (Source of data: Insight)

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Why do we study networks?

Social/Economic Networks are a way of representing interactions among units, where

- units are usually individuals/firms/countries.
- links: friendship, business relationship, communication channel

- Examples?
    - Trade Flows
    - Communication and Transportation networks
    - Diffusion of technology, knowledge
    - Credit and financial linkages

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Tool sets

Economics of networks involves

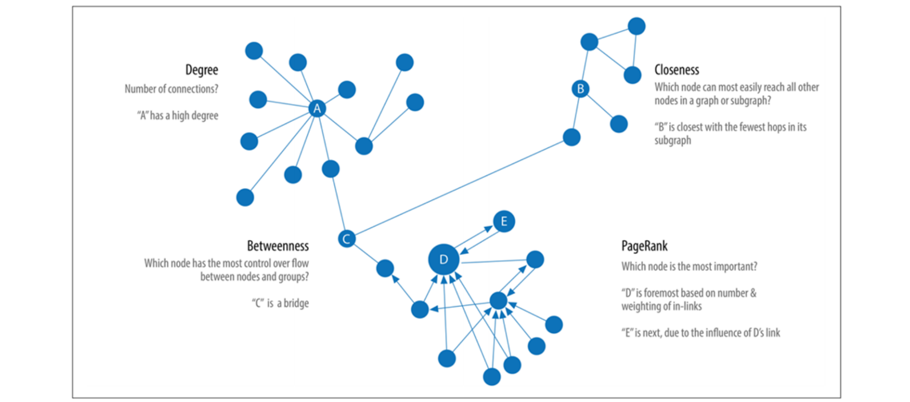
1. physical modeling of network structure (graph theory)
   - → some countries have more technology/production capabilities
   - → network serves mainly as a conduit, much of the resulting behavior can be traced directly to network structure
2. study of individual behavioral responses (game theory)
   - → the interaction between network structure and outcomes more complicated requiring some dynamic and/or equilibrium analysis
   - → eg firms adjust their behavior "strategically"
   - → even without intervention, major shock like a chip shortage, make firms adjust their behavior "strategically."
   - → strategic complementary in pricing?

Application 1: Evolution of the Smartphone Supply chain

Characteristics of the Supply Chain network

	[2009:2012]	**[2013:2016]**	**[2017:2020]**
# of different countries (nodes)	21	16	15
# of different Buyers	18	12	11
# of different Sellers	13	10	9
Number of supply links (edges)	224	154	130

How to capture relevance of individual countries in the supply chain?



Measures of Centrality

Application 2: Firms and Trade

Aggregate exports from a specific country to destination j:

$$x_j = f_j p_j b_j d_j x_j \tag{1}$$

where f_j, p_j, and b_j are # of exporters, products, and importers, respectively; $dj = \frac{o_j}{(f_j p_j b_j)}$ represent density: o_j is # of exporter–product–buyer observations for which trade with country j is positive; and $x_j = \frac{x_j}{o_j}$ is average value per exporter–product–buyer.

Table 1 The margins of trade: Norwegian aggregate exports to 205 destination countries in 2006

	Sellers	Products	Buyers	Density	Intensive margins
Exports (log)	0.57[a]	0.53[a]	0.61[a]	−1.05[a]	0.32[a]
	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)
N	205	205	205	205	205
R^2	0.86	0.85	0.81	0.81	0.50

Robust standard errors are in parentheses.

Buyer-Supplier Network

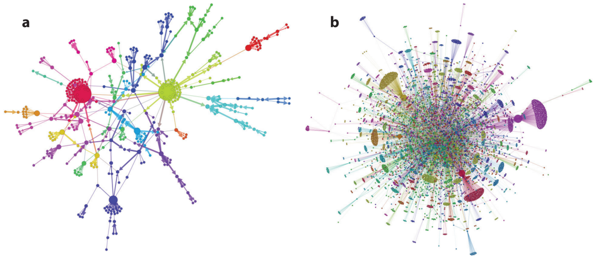


Figure 1

Firm-to-firm trade. Every node represents a firm, and the arrows show the direction of trade. The size of the node represents how many relationships a given firm has, while the size of the arrow shows the value of the transaction. (*a*) Relationships between US importers and Norwegian exporters of HS product 847990 in 2006. (*b*) Relationships between US importers and Norwegian exporters of all products in 2006.

Bernard et al. (2018)

Application 3: Social networks in labor markets

- $\rightarrow$ labor markets function efficiently
- $\rightarrow$ effects on human capital investment as well as inequality

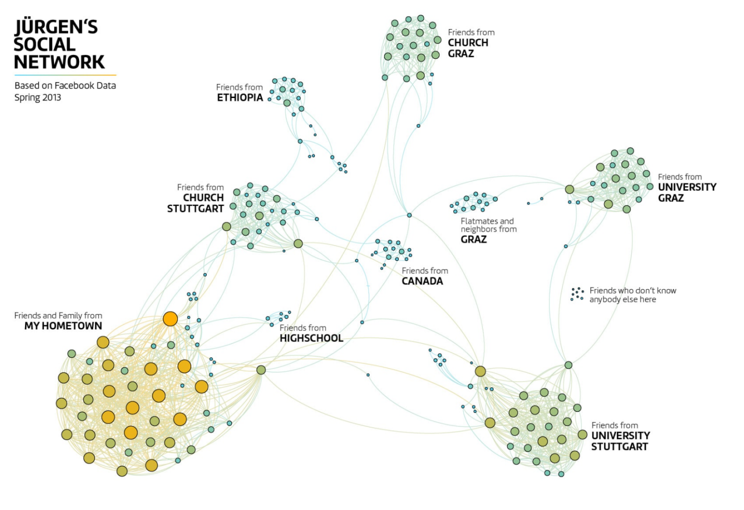
Bayer, Ross and Topa (2005): estimate following model using Census Data

$$W_{ij} = \lambda_i + \lambda_j + \beta R^b_{ij} + \epsilon_{ij} \tag{2}$$

where i, j are 2 ind. W is dummy work in the same Census block, Rij equals one if i and j reside in the same Census block.

$H_0 : \quad \beta = 0$ no local social interaction effect exists

Which type of network properties is social interaction ?



Geographical Clustering

Overview of today

- definitions
    - representation of networks
    - type of networks
- statistics to characterize a network
    - walk, path, length
    - nodes' degree and degree distribution
    - centrality measure
- Null models: random graphs
- application to economic complexity
    - Huasmann and Hidalgo etc.

Network Representation

A network is a made up of vertices (also called nodes or points) which are connected by edges (also called links or lines)

- Eg the trade network has countries as vertices and trade flows as edges

A network is typically represented by its adjacency matrix

- If nodes are indexed $i = 1, \ldots, n$ then A which is a $n \times n$ matrix where

$$A_{ij} = \begin{cases} 1, & \text{if there is an edge from j to i,} \\ 0, & \text{otherwise} \end{cases}$$

Network Types

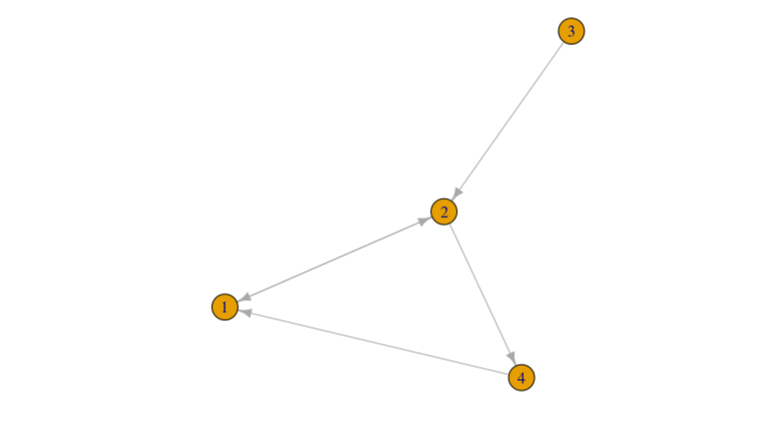
- G as a weighted graph: when the edge weight Aij takes on non-binary values (even negative) values representing the intensity of the interaction
  - e.g. share of world trade flows
- G as simple graphs: diagonal elements are zero.
- G as a un- or directed graph: directed if Aij $\neq$ Aji, and an undirected graph if Aij = Aji $\forall i, j \in N$
  - e.g. trade inflows vs outflows

Example of Directed Graph

```
> library(igraph)
> edge_list <- tibble(from = c(1, 2, 2, 3, 4), to = c(2, 3, 4, 2, 1))
> node_list <- tibble(id = 1:4)
> directed_g<- graph_from_data_frame(d = edge_list,
                         vertices = node_list, directed = TRUE)
> get.adjacency(directed_g)
4 x 4 sparse Matrix of class "dgCMatrix"
  1 2 3 4
1 . 1 . .
2 . . 1 1
3 . 1 . .
4 1 . . .
```

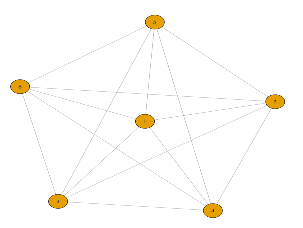
Example of Directed Graph

```
> plot(directed_g, edge.arrow.size = 0.2)
```

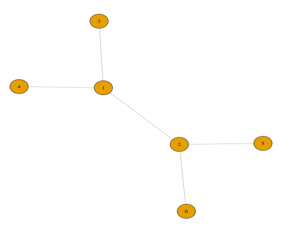


Other type of graphs

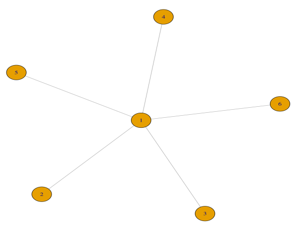
Complete Graph



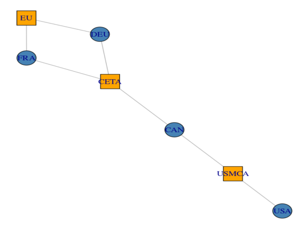
Tree



Star



Bipartite Network



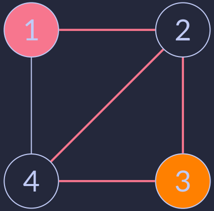
Practical Corner: Bipartite Network

```
# generate a dataframe to represents all the edges of your bipartite ntw
d <- data.frame(country=c("DEU", "DEU", "FRA", "FRA", "CAN","CAN", "USA"),
            trade_agr=c("CETA","EU", "EU", "CETA","CETA","USMCA","USMCA"))
# trasform it in a graph
g <- graph_from_data_frame(d, directed = FALSE)
# define color and shape mappings to distinguish nodes type
V(g)$label <- V(g)$name
V(g)$type <- 1
V(g)[name %in% d$trade_agr]$type <- 2
col <- c("steelblue", "orange")
shape <- c("circle", "square")
plot(g,
     vertex.color = col[V(g)$type],
     vertex.shape = shape[V(g)$type]
)
```

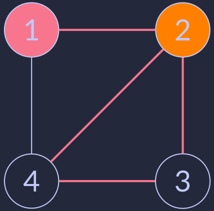
Metrics

Stats on the ”sequences of edges” informs on indirect interactions:

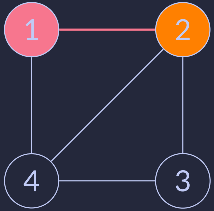
- A walk is a sequence of edges $\{i_1, i_2\}, \{i_2, i_3\}, ..., \{i_{K-1}, i_K\}$.
- A path between nodes i and j is a sequence of edges $\{i_1, i_2\}, \{i_2, i_3\}, ..., \{i_{K-1}, i_K\}$ such that i1 = i and iK = j, and each node in the sequence $i_1, ..., i_K$ is distinct
  - The length of a walk (or a path) is the number of edges on that walk (or path)
  - A geodesic between nodes i and j is a “shortest path” (i.e., with minimum number of edges) between these nodes



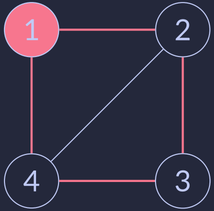
A walk



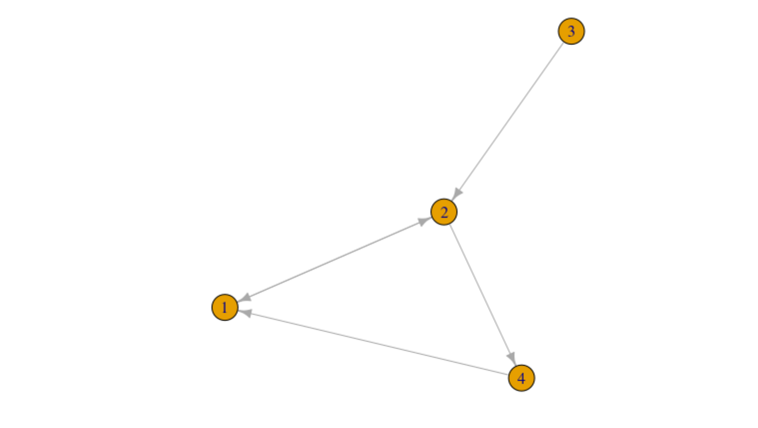
A path



Shortest Path



Cycle


3
2
1
4

Stats for graphs

```
> igraph::all_simple_paths(directed_g, 3, 1)
[[1]]
+ 3/4 vertices, named, from 2c34291:
[1] 3 2 1

[[2]]
+ 4/4 vertices, named, from 2c34291:
[1] 3 2 4 1

> igraph::shortest_paths(directed_g, 3, 1)
$vpath
$vpath[[1]]
+ 3/4 vertices, named, from 2c34291:
[1] 3 2 1
```

Features of a graph

Call $l(i,j)$ the length of the shortest path (or geodesic) between node i and j

- The maximum number of edges in a simple graph is $\binom{n}{2} = \frac{n(n-1)}{2}$
- the diameter of a network is the largest distance between any two nodes in the network: diameter $= \max_{i,j} l(i,j)$
- The average path length is the average distance between any two nodes in the network: average path length = $\frac{\sum_{i \geq j} l(i,j)}{\frac{n(n-1)}{2}}$

Environmental cooperation agreements network

Carattini et al. (2022): Countries as nodes and edges represent whether there is an environmental agreement between that country pair.

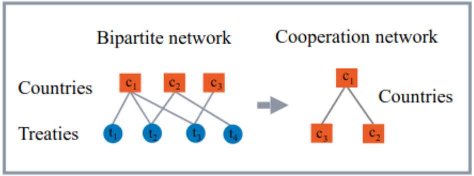
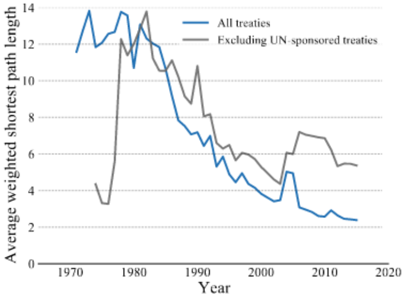


Figure 1: **Network construction**

Environmental cooperation agreements network



(b) Average shortest path length

Reference at this link

Degrees of nodes

The neighborhood of node i is the set of nodes that i is connected to

- The degree of node i is the number of edges connected to i (i.e., cardinality of his neighborhood)

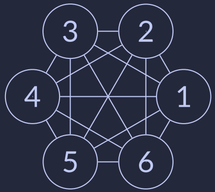
For undirected graphs:

- the degree of node i is given by $k_i = \sum_{i,j} Aij$

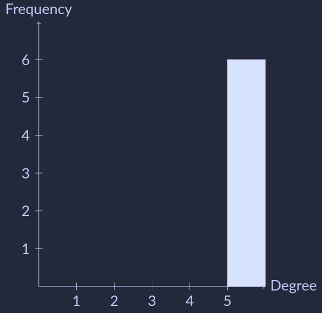
For directed graphs:

- Node i's in-degree is $\sum_j^n Aij$ (number of incoming edges)
- Node i's out-degree is $\sum_j^n Aji$ (number of outgoing edges)

Degree Distributions



Complete Graph with 6 nodes



Histogram of Degree Distribution

Degree Centrality

Captures importance of a node's position in the network. There are several possible metrics.

1. $C_i = k_i$,

- For directed networks, both in-degree and out-degree can be used as centrality measures.
- Simple, but intuitive: obs with more connections have more influence
- Does not capture "cascade effects": importance better captured by having connections to important nodes (e.g. eigenvector centrality)

Out-Degree Centrality in Trade Network 2017

```
> data_baci_y %>% head()
# A tibble: 6 × 6
      t exp       j k           v imp
  <dbl> <chr> <dbl> <chr>   <dbl> <chr>
1  2017 AFG      12 130120   5.94 DZA
2  2017 AFG      12 130190   5.12 DZA
3  2017 AFG      24 291412 14.9   AGO
4  2017 AFG      24 321511 14.2   AGO
5  2017 AFG      24 392620   1.90 AGO
6  2017 AFG      24 731512   2.05 AGO
> data_baci_y %>% select(exp,imp) %>% distinct() %>% group_by(exp) %>%
+     mutate(degree=n()) %>% select(exp, degree) %>% distinct() %>%
+     arrange(-degree)
# A tibble: 221 × 2
# Groups:   exp [221]
   exp   degree
   <chr>  <int>
 1 GBR      219
 2 ITA      218
 3 NLD      217
 4 FRA      216
 5 BEL      215
 6 DEU      214
```

In-Degree Centrality in Trade Network 2017

```
> data_baci_y %>% select(exp,imp) %>% distinct() %>% group_by(imp) %>%
+   mutate(degree=n()) %>% select(imp, degree) %>% distinct() %>%
+   arrange(-degree)
# A tibble: 221 × 2
# Groups:   imp [221]
   imp   degree
   <chr>  <int>
 1 FRA      219
 2 CZE      214
 3 GBR      214
 4 USA      213
 5 POL      212
```

Other Centrality Measures

Closeness centrality measures how close a node i is to any other node:

2. $C_i = \left(\frac{1}{n-1}\sum_{j \neq i} l_{ij}\right)^{-1}$

where $l_{i,j}$ is the shortest path between i and j.

Account for who are your neighbors: for a given number of neighbours, the more connected they are the more central you are.

Betweenness Centrality

- It is based on the concept of shortest paths between pairs of nodes.
- In a social network, a node with high betweenness centrality acts as a bridge, connecting different communities or groups.

The betweenness centrality of a node v is calculated as the fraction of shortest paths between all pairs of nodes that pass through that particular node.

3.

$$C_B(v) = \sum_{i \neq j \neq v} \frac{\sigma_{ij}(v)}{\sigma_{ij}}$$

where σ_{ij} is the total number of shortest paths from node i to node j, and $\sigma_{ij}(v)$ is the number of those paths that pass through node v.

In a friendship network: What is degree centrality?
would correspond to who is the most popular kid.
Closeness centrality?
would correspond to who is closest to the rest of the group, so this would be relevant if we wanted to understand who to inform or influence for information to spread to the rest of the network
Betweenness ?
would be relevant if the thought experiment was which individuals would have to be taken out of the network in order to break the network into separate clusters

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Centrality Measures

```
> g <- data_baci_y %>% select(exp,imp) %>% distinct() %>%
+   graph_from_data_frame(., directed = FALSE)
> setNames(rownames_to_column(data.frame(closeness(
+   g,
+   vids = V(g),
+   mode = c("out"),
+   weights = NULL,
+   normalized = TRUE))), c("country", "centrality")) %>%
+   arrange(-centrality)  %>%
+   head()
  country centrality
1     FRA  1.0000000
2     ITA  0.9954751
3     GBR  0.9954751
4     BEL  0.9909910
5     CZE  0.9909910
```

Introduction

- Random graph null models are used to compare real-world networks against randomized versions.
- They help identify the presence of structural properties or patterns in the observed network.
- Null models provide a baseline for testing hypotheses about network properties.

Randomization Methods

1. **Edge Rewiring:** Randomly rewire edges while preserving the degree distribution.
2. **Degree Preserving:** Randomly shuffle node labels while preserving the degree sequence.
3. **Configuration Model:** Generate a random graph with the same degree sequence as the observed network.
4. **Erdős-Rényi Model:** Generate a random graph with a fixed number of nodes and edges.

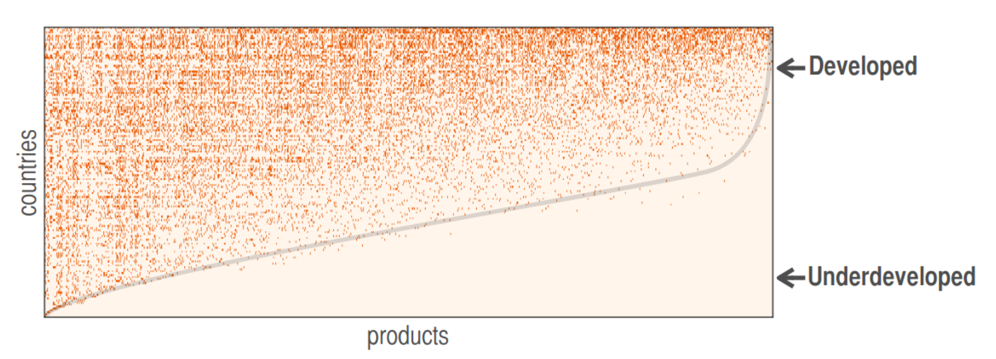
Comparing Network Statistics

- Compute network statistics (e.g., clustering coefficient, degree distribution, etc.) for the observed network.
- Generate multiple random graph null models.
- Calculate the same network statistics for each null model.
- Compare the observed network statistics against the null model distributions.
- Assess whether the observed network statistics significantly deviate from the null model distributions.

APPLICATIONS

"The productivity of a country resides in the diversity of its available non-tradable capabilities, and therefore, cross-country differences in income can be explained by differences in economic complexity, as measured by the diversity of capabilities present in a country and their interactions."
Hidalgo and Hausmann 2009

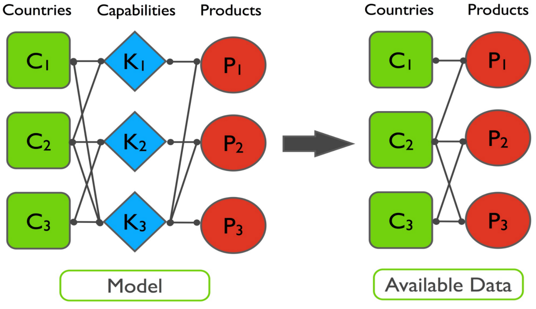
Matrix of diversification of countries



Source: Cristelli, Tacchella, Pietronero (2014)

The theory of hidden capabilities

A country is able to produce a product when it has the capabilities to do it (Hausmann & Hidalgo 2009)



Source: Hidalgo et al. (2009)

Network structure

Let us index countries with $c = 1, \ldots, n$ and products with p
The bipartite network is represented by means of a bi adjacency matrix B of size $n \times p$

$$B_{cp} = \begin{cases} 1, & \text{if country } c \text{ is a significant exporter of the product } p, \\ 0, & \text{otherwise} \end{cases}$$

Significant exporter, when

$$RCA_{cp} = \frac{\frac{q_{cp}}{\sum_p q_{cp}}}{\frac{\sum_c q_{cp}}{\sum_c \sum_p q_{cp}}} > 1 \tag{3}$$

which is whenever the share of product p in the country export basket is larger than its share in the world trade

Method of Reflections

MoR consists of iteratively calculating the average value of the previous-level properties of a node's neighbors and is defined as the set of observables:

$$k_{c,N} = \frac{1}{k_{c,0}} \sum_p B_{cp}\, k_{p,N-1}$$

$$k_{p,N} = \frac{1}{k_{p,0}} \sum_c B_{cp}\, k_{c,N-1}$$

for $N \geq 1$. With initial conditions given by the degree, or number of links, of countries and products, $k_{c,0} = \sum_p B_{cp}$ (diversification) and $k_{p,0} = \sum_c B_{cp}$ (ubiquity)

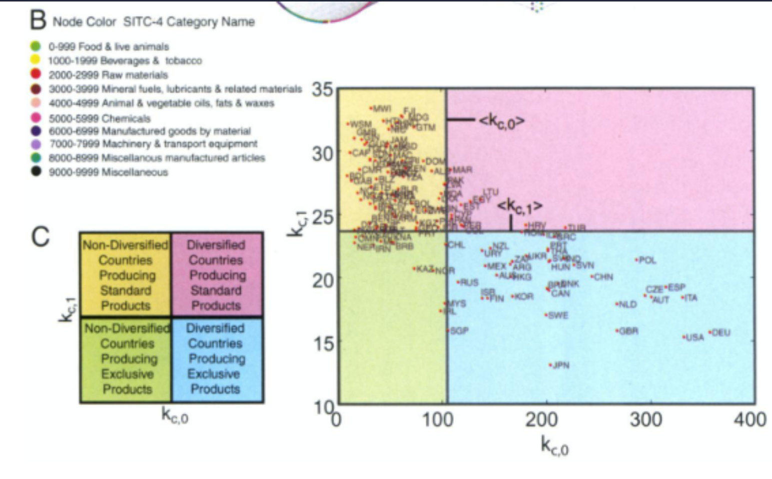
Methods of Reflections

Definition	Working Name	**Description:** Short summary Question Form
$k_{a,0}$	Diversification	Number of products exported by country a. How many products are exported by country a?
$\kappa_{\alpha,0}$	Ubiquity	Number of countries exporting product α. How many countries export product α?
$k_{a,1}$	$k_{c,1}$	Average ubiquity of the products exported by country a. How common are the products exported by country a?
$\kappa_{\alpha,1}$	$k_{p,1}$	Average diversification of the countries exporting product α. How diversified are the countries that export product α?
$k_{a,2}$	$k_{c,2}$	Average diversification of countries with an export basket similar to country a How diversified are countries exporting goods similar to those of country a?
$\kappa_{\alpha,2}$	$k_{p,2}$	Average ubiquity of the products exported by countries that export product α. How ubiquitous are the products exported by product's α exporters?

Table S 1 Interpretation of the bipartite network description obtained from the method of reflections.

For countries, even variables (kc,0 ,kc,2 ,kc,4,...) are generalized measures of diversification, whereas odd variables (kc,1 ,kc,3 ,kc,5,...) are generalized measures of the ubiquity of their exports.

Results



Source: Hidalgo et al. (2007)

Null Model

They construct two random matrices

- availability of capabilities (a)

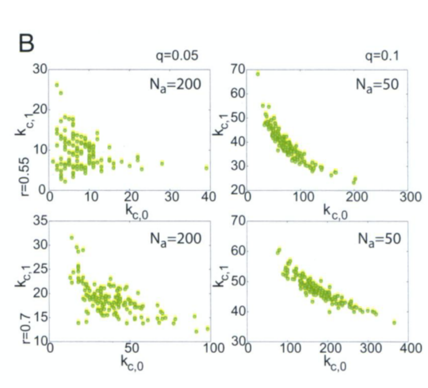
$$C_{ca} = \begin{cases} 1, & \text{with prob. r} \\ 0, & \text{with prob 1-r} \end{cases}$$

- necessary capabilities to produce products

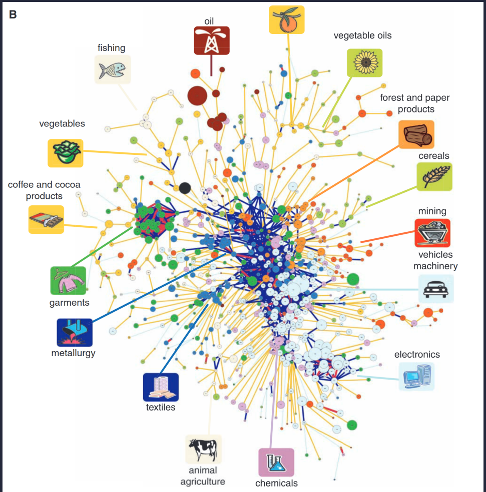
$$\Pi_{pa} = \begin{cases} 1, & \text{with prob. q} \\ 0, & \text{with prob 1-q} \end{cases}$$

$\hat{B}_{cp} = 1$ if $\sum_a \Pi_{pa} = \sum_a \Pi_{pa} C_{ca}$, 0 otherwise.

Results of the null model

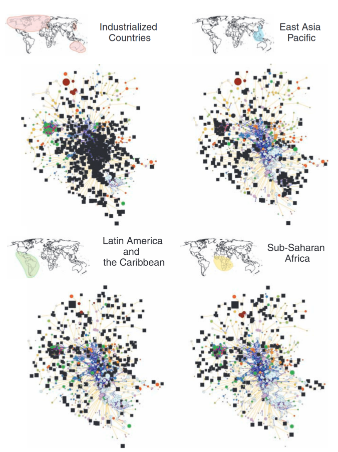


The Product Space of Trade



Source: Hidalgo et al. (2007)

Countries in the Product Space



Source: Hidalgo et al. (2007)

Applications:

A list of papers that address the following questions can be found in this work:

- Role of Demand Externality
- How are education and other human capital decisions influenced by social network structure?
- Will the networks that are formed be the efficient ones in terms of their implications for economic activity?

Sources

- Jackson, Matthew O. Social and economic networks. Vol. 3. Princeton: Princeton university press, 2008.
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